

Coherent Directed Transport of Neutral Atoms in Unmodulated Optical Lattices

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We introduce a fully coherent way for directed transport of neutral atoms in optical lattices by regularly performing phase shifts on the lattice potential during the free evolution of the system. This paves the way for realizing quantum gates and entanglement generation by bringing two individual atoms in the proximity of each other and let them interact. The speed of our protocol is truly determined by the tunneling amplitudes of the atoms and thus is much faster than the speed of the dynamics resulted from superexchange interaction in spin chains or renormalized tunnelings in AC-driven quantum ratchets. We also introduce an entanglement witness which allows for experimentally detecting entanglement through local rotations and measurements. Our scheme is robust against possible imperfections and perhaps its main advantage is its simplicity which does not need sophisticated cooling mechanism and all its requirements have been already achieved in recent experiments.

Introduction.— Mediating interaction between arbitrary pairs in a network of qubits is essential for realizing two-qubit entangling gates and thus for universal quantum computation [1]. Unlike the neighboring qubits, interacting the distant ones is notoriously difficult. One way to mediate interaction between them is to use an additional setup, called quantum bus. Spin chains are the most common quantum buses where their evolution realize quantum gates between distant points [2–5]. However, to avoid their dispersive dynamics one has to either delicately engineer the couplings [2, 3] or switch to the perturbative regimes [4, 5] which are either difficult to fabricate or suffer from slow dynamics. To have both the flexibility and the speed one may think of using mobile qubits as interaction mediators [6]. This invokes the *directed transport* of one or more “localized” qubits above a network of registers. Directed transport in the absence of an external biased force in a periodic structure, known as ratchet effect, usually depends on a dissipative process in the system [7]. In the absence of dissipation, one may exploit the AC modulation of a periodic potential to renormalize the tunneling amplitudes [8] which can be used for controlled transport of atoms [9] or quantum states [10] in a lattice. Unfortunately, realizing the AC-driven transport for *localized individual* atoms demands a complicated setup and due to its slow dynamics practically has no benefit over spin chains. One may use optimal control theory to improve the speed of the AC-driven transport [11] but then the sensitivity to the optimized pulse and its practicality become a new obstacle. So, one may ask whether the coherent evolution of an unmodulated system can transport physical qubits from one point to another through a fast dispersionless process?

Cold atoms in optical lattices is now a very fast developing field. Generating a Mott insulator phase, with exactly one atom per site, is an ordinary experiment now [12] which enables for realizing effective spin Hamiltonians [13] via superexchange interaction. Since, this is a second order process the effective spin couplings are $4J^2/U$ (where, J is the tunneling and U is the onsite energy). These weak couplings not only result in very slow dynamics for spin chains in opti-

cal lattices [3, 8, 14] but also demand very low temperatures for observing magnetic phases [3] and despite being at the edge of experimental achievements they have not yet been realized [15]. To avoid the slow superexchange process, new experiments exploiting the direct tunneling of particles, which are controlled by J instead of $4J^2/U$, have been developed for observing controlled tunneling [8] and simulating antiferromagnetism [16]. Moreover, single site resolution in current experiments [17, 18] allows for single qubit operations and measurements [18–20]. A series of multiple two-qubit gates, acting globally and simultaneously, has been also realized [21] between neighboring atoms. Unfortunately, this gate mechanism cannot be used for distant qubits as in this approach, atoms become delocalized and thus dephasing restricts the distance to ~ 10 sites [22]. These show that mediating interaction between *individual distant* atoms is still an open problem for optical lattice experiments.

In this letter, we put forward a *simple* way for directed transport of neutral atoms in optical lattices through regularly performing phase shifts on the lattice potential during the free evolution of the system. Unlike the AC-driven transport [8–10] and spin chain gate realizations [3, 14] the dynamics is truly governed by the tunneling itself and thus is fast and does not need extreme low temperatures. Furthermore, we develop an entanglement witness which can be used for experimentally verifying entanglement. The importance of our proposal indeed lies in its simplicity which makes it *timely and fully accessible* to current experiments. We also show its robustness against possible imperfections in an experimental realization.

Model.— We consider an optical superlattice made of an standing wave formed by two different set of counter propagating laser beams where one frequency is twice the other. The resulting potential is

$$V(x) = V_l \cos^2(2\pi x/\lambda_l) + V_s \cos^2(2\pi x/\lambda_s + \phi) \quad (1)$$

where, $\lambda_l = 2\lambda_s$ are the wave lengths, V_l and V_s are the amplitudes and ϕ is the phase shift between laser beams. The low

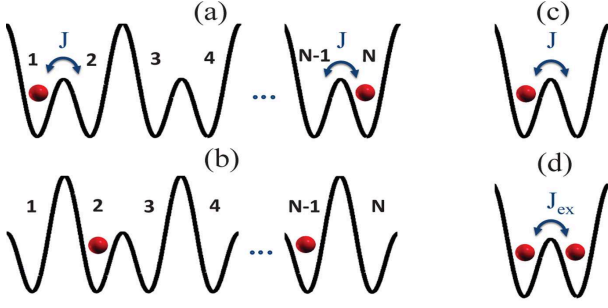


FIG. 1: (Color online) (a) A superlattice containing two atoms in sites 1 and N with tunnelings J . (b) Applying a phase shift, after accomplishing the tunneling, changes the lattice potential and exchanges the odd and even tunnelings. (c) Dynamics of a single atom, governed by tunneling J , in a double well. (d) Superexchange interaction with strength J_{ex} between two atoms in a double well.

energy Hamiltonian of atoms trapped by $V(x)$ is [13]

$$H = - \sum_{\langle i,j \rangle, \sigma} (J_i a_{i,\sigma}^\dagger a_{j,\sigma} + H.C.) + \frac{U}{2} \sum_{i,\sigma} n_{i,\sigma} (n_{i,\sigma} - 1) + \frac{U}{2} \sum_i n_{i,\uparrow} n_{i,\downarrow}, \quad (2)$$

where, $\langle i, j \rangle$ denotes the nearest neighbor sites, $a_{i,\sigma}$ annihilates one atom with spin $\sigma = \uparrow, \downarrow$ at site i , and $n_{i,\sigma} = a_{i,\sigma}^\dagger a_{i,\sigma}$. The tunneling J_i 's are controlled by the amplitudes V_l and V_s such that all even (odd) couplings take the same amplitude J_e (J_o). Changing the phase shift ϕ from 0 to $\pi/2$ displaces the lattice potential by λ_s and thus exchanges the even and odd couplings. We assume that the amplitudes V_l and V_s are tuned such that we get $J_o = J$ and $J_e = 0$ for $\phi = 0$ (see Fig. 1(a)) and $J_o = 0$ and $J_e = J$ for $\phi = \pi/2$ (see Fig. 1(b)). This choice of couplings make the system a series of decoupled double wells which their dynamics are fully independent. A schematic picture of the system is depicted in Figs. 1(a) and (b) in which atoms in double wells are evolved under the action of their own double well Hamiltonian.

Directed transport.— Here we consider a superlattice potential with $\phi = 0$ for which $J_o = J$ and $J_e = 0$. As the result, a single atom localized at a single site, is subjected to a double well potential as shown in Fig. 1(c). We denote the state of the atom localized in the left (right) site as $|\sigma, 0\rangle$ ($|0, \sigma\rangle$) where, σ represents the spin of the atom and 0 denotes an empty site. The evolution of these states in a double well are given by

$$\begin{aligned} |\sigma, 0\rangle &\rightarrow \cos(Jt)|\sigma, 0\rangle + i \sin(Jt)|0, \sigma\rangle \\ |0, \sigma\rangle &\rightarrow \cos(Jt)|0, \sigma\rangle + i \sin(Jt)|\sigma, 0\rangle. \end{aligned} \quad (3)$$

At an optimal time $t_h = \frac{\pi}{2J}$, atom is completely hopped to the other site leaving its initial site empty. When tunneling is completed after $t = t_h$, by performing a phase shift $\phi = \pi/2$ to the lattice potential, all the large (small) barriers in the potential are replaced by small (large) ones, as shown in Fig. 1(b), and atom thus resides in a new double well. So, the dynamics of Eq. (3) is repeated and atoms hops one site

further. Repeating this process for $N - 1$ times transports the atom from site 1 to site N and with choosing the initial phase ϕ one can determine the direction of transport towards the right or left. It is also worth to mention that according to Eq. (3) at each hopping step, the quantum state of the system gets an overall phase but as this phase is independent of the internal state σ it can be treated as a global phase which has no effect and thus we ignore it in the rest of this paper.

One may use the above mechanism to transport two or more atoms simultaneously. In the case that both atoms sit in even or odd sites they hop together in the same direction and thus never reach each other. To bring them together one can tilt the lattice symmetrically around the mid point of the atoms by applying a gradient magnetic fields such that both atoms sit symmetrically in opposite sides of the maximum of the tilted optical lattice potential. With this geometry, one particle can tunnel down to its neighboring site while the other cannot due to a higher potential. This mechanism moves one of the atoms by one site and then by restoring the tilted lattice to its initial form atoms can be transported towards each other.

Two-qubit gate and entanglement generation.— When two atoms, initially located at a distance in an optical lattice, are directed to the same double well, they can interact directly and may get entangled there. In Fig. 1(d) we schematically show two atoms in different sites of the same double well interacting through Hamiltonian H . The Hilbert space of two atoms in a double dot is spanned by $|\sigma, \sigma'\rangle$, $|0, \sigma\sigma'\rangle$ and $|\sigma\sigma', 0\rangle$ where, 0 represents an empty site, $\sigma, \sigma' = \uparrow, \downarrow$ and two sites are separated by “,”. The evolution can be written as

$$\begin{aligned} |\sigma, \sigma\rangle &\rightarrow A_{\sigma,\sigma}(t) |\sigma, \sigma\rangle + C_{\sigma,\sigma}(t) (|\sigma\sigma, 0\rangle + |0, \sigma\sigma\rangle) \\ |\sigma, \sigma'\rangle &\rightarrow A_{\sigma,\sigma'}(t) |\sigma, \sigma'\rangle + B_{\sigma',\sigma}(t) |\sigma', \sigma\rangle + \\ &C_{\sigma,\sigma'}(t) (|\sigma\sigma', 0\rangle + |0, \sigma\sigma'\rangle), \text{ for } \sigma \neq \sigma' \end{aligned} \quad (4)$$

where,

$$\begin{aligned} A_{\sigma,\sigma}(t) &= \cos(sUt/2) + \frac{i}{s} \sin(sUt/2) \\ C_{\sigma,\sigma}(t) &= i \frac{2\sqrt{2}J}{sU} \sin(sUt/2) \\ A_{\sigma,\sigma'}(t) &= \frac{e^{iUt/4}}{2} + \frac{1}{2} (\cos(s'Ut/4) + \frac{i}{s'} \sin(s'Ut/4)) \\ B_{\sigma,\sigma'}(t) &= -\frac{e^{iUt/4}}{2} + \frac{1}{2} (\cos(s'Ut/4) + \frac{i}{s'} \sin(s'Ut/4)) \\ C_{\sigma,\sigma'}(t) &= \frac{i4J}{s'U} \sin(s'Ut/4), \end{aligned} \quad (5)$$

for $s = \sqrt{1 + 16J^2/U^2}$ and $s' = \sqrt{1 + 64J^2/U^2}$. This dynamics may look very complicated as doubly occupied sites are also involving but in the limit of $J \ll U$, where the double occupancy is negligible and Hamiltonian is effectively an XX spin Hamiltonian with superexchange coupling $J_{ex} = 4J^2/U$ [13], it takes a simple form (up to a global phase) as

$$\begin{aligned} |\sigma, \sigma\rangle &\rightarrow |\sigma, \sigma\rangle \\ |\sigma, \sigma'\rangle &\rightarrow \cos(J_{ex}t) |\sigma, \sigma'\rangle + i \sin(J_{ex}t) |\sigma', \sigma\rangle. \end{aligned} \quad (6)$$

This dynamics explains a conditional gate acting on two qubits and in particular after interacting for $t_I = \frac{\pi}{2J_{ex}}$ it generates a maximally entangled state from a fully separable initial

state $|++\rangle$ where, $|+\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. When interaction is accomplished one may separate two atoms from each other and bring them back to their initial positions by the directed transport mechanism explained above.

The most important issue in the phase gate operation is that U should be large to avoid double occupancy but at the same time it cannot be that large as the interaction time t_I increases by increasing U . Our simulations and recent experiments suggest an intermediate value of $U \simeq 25J$. It is important to notice that in the whole process of bringing the atoms together, entangle them and then return them to their initial position there is only one step in which interaction is governed by superexchange coupling J_{ex} and the rest is controlled by J . This makes our entangling process faster than spin chains [3, 14] where the whole process is governed by J_{ex} .

Entanglement detection.— Local unitary operations on individual atoms [17, 18] and single spin measurement [20], both achieved recently in optical lattices, allow for experimental detection of entanglement. Here, we introduce an entanglement witness which its average value, computable through local measurements, distinguishes between the entangled and separable states. An entanglement witness is a Hermitian operator which has negative average value for entangled states and positive for separable states. So, one can repeatedly measure the entanglement witness in the lab and compute its average value and verify the presence of entanglement. Following the footsteps of the Ref. [23] we find the following witness operator for detecting entanglement between qubits 1 and N

$$W = \frac{1}{2}\{I - Z_1 Y_N - Y_1 Z_N - X_1 X_N\}, \quad (7)$$

where X_k , Y_k and Z_k are Pauli operators acting on qubit k and I is identity operator. For spin measurements one has to shine an intense laser beam localized on a particular site and vertical to the chain axis [20] which may restrict the measurement only to the z direction. So, to end up with only z measurements in the witness operator one has to apply some local unitary operators to convert X and Y Pauli operators, appeared in W , to Z . This indeed can be done as $X = hZh$ and $Y = e^{i\frac{\pi}{4}X} Z e^{-i\frac{\pi}{4}X}$ where, $h = (X+Z)/\sqrt{2}$ is the Hadamard gate. In fact, in an experimental setup each term in W should be measured in a separate measurement and with the means of these local unitary operators, achievable by shining localized laser beams [17, 18], each term simplifies to a measurement in the z direction. Since the optical lattice is almost empty we do not need very localized laser beams for unitary operations which makes our setup even simpler for physical realization.

Time scale.— The total time needed for bringing two atoms together from the distance N , let them interact to get entangled and then restore them back in their initial positions is $t_T = (N-2)t_h + t_I$. By considering the values of t_h and t_I one reads $t_T = \frac{\pi(N-2)}{2J} + \frac{\pi U}{8J^2}$. This time scale is much faster than those from spin chains which scale as $\sim NU/4J^2$ [3, 14]. To have some estimations for t_T we take the parameters of a very recent experiment [8] where, $J/h \simeq 1.5$ KHz and $J_{ex}/h \simeq 250$ Hz (these give $U \simeq 25J$). For a distance

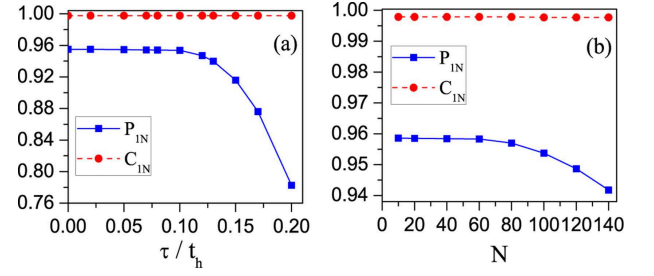


FIG. 2: (Color online) (a) P_{1N} and C_{1N} versus switching time τ/t_h for a chain of length $N = 100$. (b) P_{1N} and C_{1N} versus distance N for the switching time of $\tau = 0.1t_h$ (in both figures $U = 25J$).

of $N = 100$, taking these values gives us $t_T \simeq 17$ ms which is well below the typical decoherence time of the internal levels ($\simeq 10$ minutes [24]). Moreover, with the same parameters using the proposal of Ref. [14] for entanglement distribution through superexchange interaction over 100 sites one gets the time scale of ~ 100 ms which is slower than our mechanism by at least a factor of 5.

Imperfections.— In any experiment there might be some imperfections for realizing a theoretical idea. For the above proposal, the first imperfection effect is double occupancy where two atoms occupy the same site. When the initial state of two atoms in a double well is $|++\rangle$ this probability indeed becomes $P_{doub} = (|C_{\uparrow\uparrow}|^2 + |C_{\downarrow\downarrow}|^2 + |C_{\uparrow\downarrow}|^2 + |C_{\downarrow\uparrow}|^2)/4$. A typical value of $U = 25J$, used in a recent experiment for superexchange interaction [8], gives $P_{doub} < 0.02$ which guarantees that the double occupancy is negligible.

So far, we have assumed that applying the phase shift ϕ is instantaneous however, it is not true and in real experiments it takes some time to perform this phase shift [21, 22]. To see this effect we assume that when odd (even) couplings, i.e. J_0 (J_e), change from J (0) to 0 (J) this transition is linear in time and happens over a time period of τ . Similarly, when odd (even) couplings change from 0 (J) to J (0) we assume a linear rise (fall) over the same time period τ . The gradual switching of the couplings couples different double wells along the lattice and delocalizes the atomic wave functions which has two main effects: (i) atomic wave functions defuse over several sites and thus atoms may not reach their destination with certainty; (ii) entanglement generation may be affected as atoms may not both be localized in the same double well to interact. To quantify these effects in the presence of gradual switching of the couplings we assume that two atoms in the state $|++\rangle$, initially localized in sites 1 and N , are directed towards each other to interact and generate entanglement. At t_T when we expect to have both atoms returned to their initial positions in a maximally entangled state we compute P_{1N} defined as the probability of finding both atoms at their initial locations by projecting the whole quantum state on to the subspace of two particles at sites 1 and N . This projection can be done experimentally by taking a fluorescent picture of the lattice at $t = t_T$ [17, 18] and P_{1N} describes the probability of finding the atoms at sites 1 and N . Then for this projected

state one can compute the concurrence C_{1N} as a measure of entanglement [25] between two atoms. To have an estimation of τ needed for applying the phase shift we take the results of Ref. [22] in which our desired phase shift is performed over the time of $\tau \simeq 15 \mu\text{s}$. For a typical value of the tunneling such as $J/h = 1.5 \text{ KHz}$ reported in [8] we get $\tau/t_h < 0.1$. In Fig. 2(a) we plot both P_{1N} and C_{1N} in terms of τ/t_h for a chain of length $N = 100$ where, we have $N - 1$ gradual switching of the couplings. As it is clear from the figure, there is plateau up to $\tau \simeq 0.1t_h$ and beyond that even for a very pessimistic value of $\tau/t_h = 0.15$ we have $P_{1N} > 0.9$. Entanglement is even more robust and remains almost one for all values of τ which shows that whenever atoms return to their desired positions entanglement is almost perfectly achieved. In Fig. 2(b) we plot P_{1N} and C_{1N} versus the distance N when $\tau = 0.1t_h$. This shows that system has a very good scalability as $P_{1N} \simeq 0.94$ for a distance of $N = 140$.

Comparison with AC-driven transport.— Directed transport may be also achieved through modulating the amplitude of an optical lattice potential with an extra AC-field [7–10]. In the limit of high frequency AC-field, the driven system behaves like the undriven one but with renormalized tunneling amplitudes [26]. In fact, for a sinusoidal driving with amplitude K and frequency ω the renormalization factor is $\mathcal{J}_0(Kx/\omega)$ where, \mathcal{J}_0 is the zeroth Bessel function and x is the distance between the two subsequent minima of the lattice. To keep atoms localized during the transportation one has to switch off the even and odd couplings alternatively [9, 10], as we also did in the above proposal. This means that one has to prepare a superlattice with different distances x_1 and x_2 between alternating minima [9, 10] which demands at least three harmonics for generating the superlattice itself. The AC driving then can switch off the even tunnelings by tuning the parameters such that $\mathcal{J}_0(Kx_1/\omega) = 0$ and thus atoms can only hop by odd couplings which are also normalized by $\mathcal{J}_0(Kx_2/\omega)$. This normalization is small unless for very distinctive values of x_1 and x_2 which is very hard (if not impossible) to achieve in potential formed by only three harmonics. Consequently, AC-driving transport not only demands for at least three laser frequencies but also suffers from a slow evolution which practically shows no benefit over spin chains.

Conclusion.— We introduced a fully coherent and easily realizable method for directed transport of neutral atoms in optical superlattices based on performing phase shifts regularly during the free evolution of the system. The importance of our proposal is its *simplicity* where all its requirements have been already achieved. In contrast to spin chains realized by optical lattices, where the superexchange couplings give a transport time of $\sim NU/J^2$ [3, 14], our coherent mechanism is fast, working within the time scale of $\sim N/J$, and does not need sophisticated cooling mechanisms for reaching extremely low temperatures. In compare to AC-driving proposals [9, 10], our scheme is simpler for practical realizations and our time scale is much faster as our proposed evolution is truly determined by the tunneling amplitudes rather than the small modulated tunnelings in AC-driven lattices [9, 10]. An immediate ap-

plication of our directed transport is to generate entanglement between distant atoms by bringing them together and let them interact. Furthermore, we proposed an entanglement witness which its average value can be computed experimentally for verifying the presence of entanglement. Finally, we showed that our scheme is robust against possible imperfections and scales very well for distances up to hundreds of sites.

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